Scattering Theory of Particles Elastic and Inelastic Scattering

- Scattering process said to be elastic if total K.E. of all particles before scattering is equal to Total K.E after scattering
- Elastic scattering occurs when there is no net conservation of K.E. into other form.
- Elastic scattering K.E as well moment is conserved.
- Consider two particles of mass m_1 and m_2 let U_1 and U_2 be the velocities of masses m_1 and m_2 before collision and v_1 and v_2 velocities after collision.
- Total momentum before collision must be equal to total moment after collision.
- $m_1U_1=m_2U_2 = m_1v_1=m_2v_2$ conservation of $\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2$
- Elastic scattering occurs when there is no loss of energy of the incident primary electron.
- Elastically scattered electrons can change direction but do not change their wavelength.

- Inelastic scattering occurs when there is an interaction that causes loss of energy of the incident primary electron. Inelastically scattered electrons have a longer wavelength.
- Inelastic scattering occurs by many mechanisms, and the energy is transferred to the specimen generating a range of useful signals that are exploited to characterize the material.
- For elastic scattering in particle physics, no energy is absorbed by either particle during the collision.
- For inelastic scattering, one or both of the particles involved in the collision absorb some or all of the original kinetic energy.
- Examples might be where an atom (target) might become ionized during / after a collision by an electron or alpha particle (or other projectile particle).
- A complex molecule (target) might start rotating or vibrating during / after a collision with an electron or alpha particle (or some other incident particle). The target atom / molecule may simply become excited (electronically) during the collision. In such cases, some of the initial kinetic energy is transferred into other types of energy. Cheers.

Inelastic Scattering

Inelastic scattering occurs when there is an interaction that causes loss of energy of the incident primary electron. Inelastically scattered electrons have a longer wavelength

Scattering Theory of Particles Elastic and Inelastic Scattering

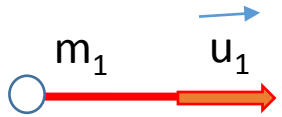
- Elastic scattering occurs when there is no loss of energy of the incident primary electron.
- Elastically scattered electrons can change direction but do not change their wavelength. Coherent elastic scattering produces the effect of electron diffraction which is used to analyze crystal structure.
- Inelastic scattering occurs when there is an interaction that causes loss of energy of the incident primary electron. Inelastically scattered electrons have a longer wavelength.
- Inelastic scattering occurs by many mechanisms, and the energy is transferred to the specimen generating a range of useful signals that are exploited to characterize the material.
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Elastic Scattering Laboratory and Centre of Mass System.

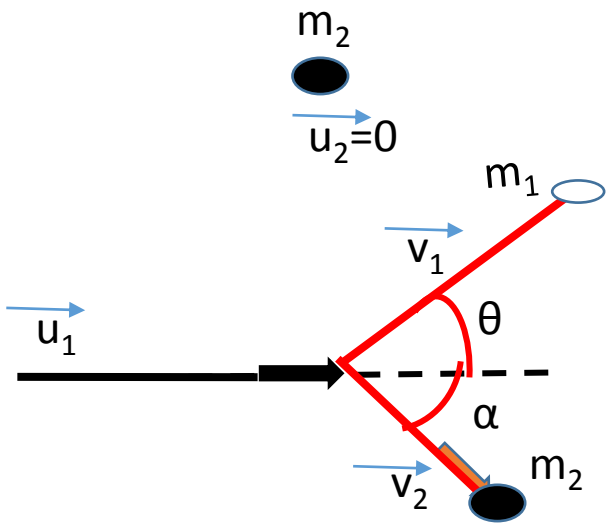
- The Scattering of particle can be Visualized 2 kind of coordinate **Laboratory and Centre of Mass System**
- **basically** Scattering theory is divided into 2 type **Classical Scattering theory and Quantum Scattering theory**

  in case of Classical Scattering theory we have **target (solid liquid, gases)** and particle is incident on it
- while in case **Quantum Scattering theory** plane wave incident on target is potential V(r).

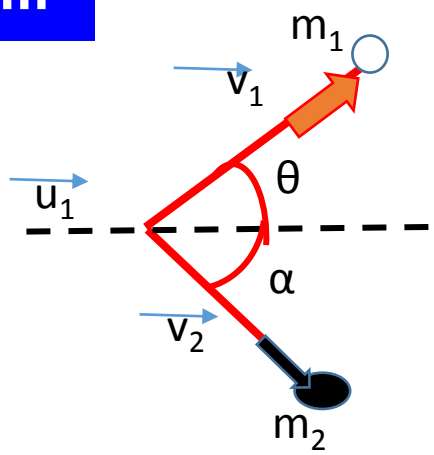
Elastic Scattering Laboratory System



Before Collision

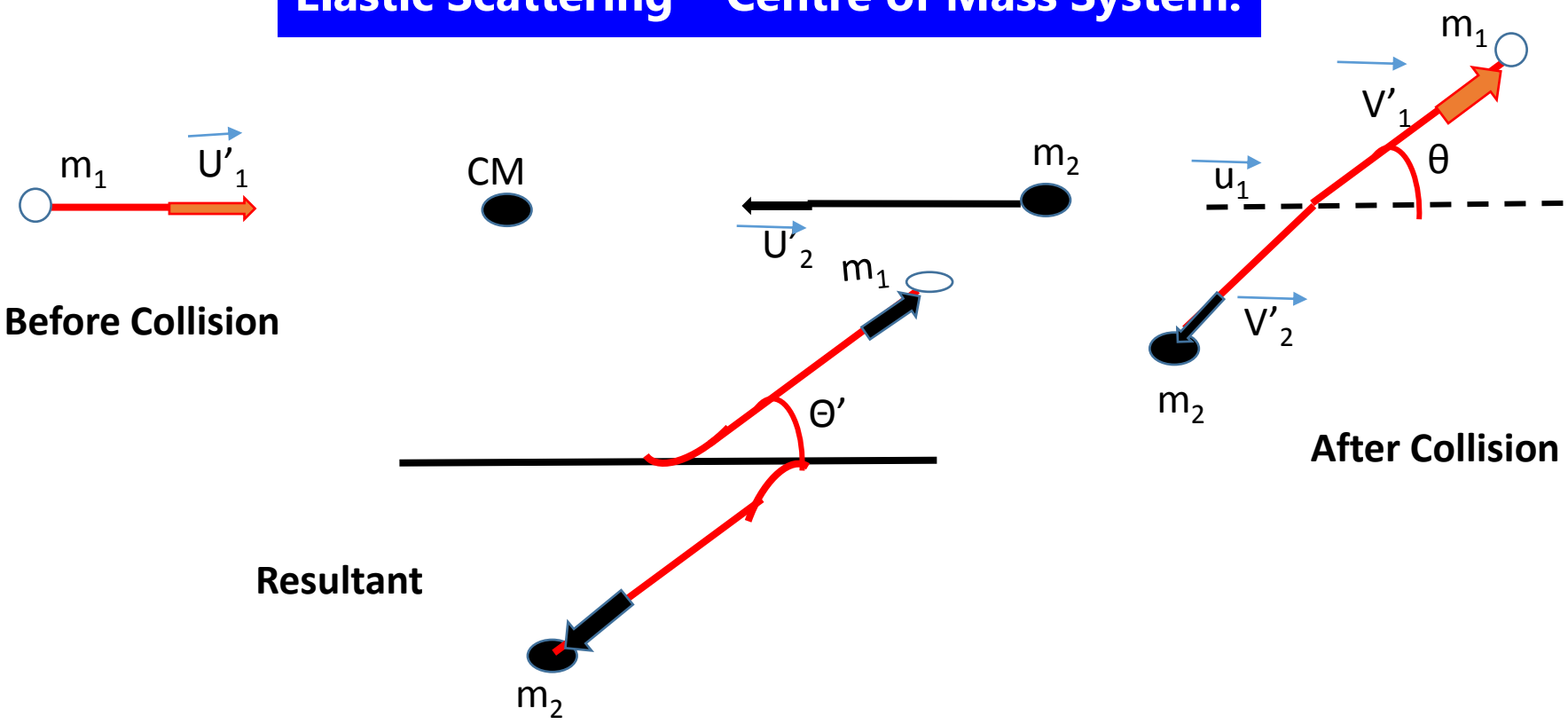


Resultant



After Collision

Elastic Scattering Centre of Mass System.



- The collision between 2 bodies in C.M system is observed when **C.M is at rest** such coordinate system referred as Centre of mass system.
- **basically** Scattering theory is divided into 2 type **Classical Scattering theory and Quantum Scattering theory** in case of Classical Scattering theory we have **target (solid liquid, gases)** and particle is incident on it
- while in case **Quantum Scattering theory** plane wave incident on target is potential V(r)

we..know..position..C.M.is..at..rest.**Before...collistion,**

$m_1\vec{u}_1' + m_2\vec{u}_2' = 0 - - - (1)$

$m_1\vec{u}_1' = -m_2\vec{u}_2'$

where..$\vec{u}_1'$ and..$\vec{u}_2'$..are..velo..and..m_1..and..m_2.masses**Before.Collistion**

After **.collistion**...$m_1\vec{v}_1' + m_2\vec{v}_2' = 0 - - - (2)$

$m_1\vec{v}_1' = -m_2\vec{v}_2'$

where..$\vec{v}_1'$ and..$\vec{v}_2'$..are..velo..and..m_1..and..m_2.masses..**After.collistion**

- In.the..C.M.frame..momenta.

of..2..particles..are..equal.in..Mag..and..opposite..in..direction

- θ..is..angle..of..scattering..in.C.M.frame.
- In..Lab.frame..momentum..of.C.Mass.is..non..zreo.

C.M.frame.it..is..equal.tozero...

let..$\vec{r}_1, \vec{r}_2$..and..$\vec{R}$..be..the..postion..vector..1..and..2..particle..their..C.M..in.Lab.frame

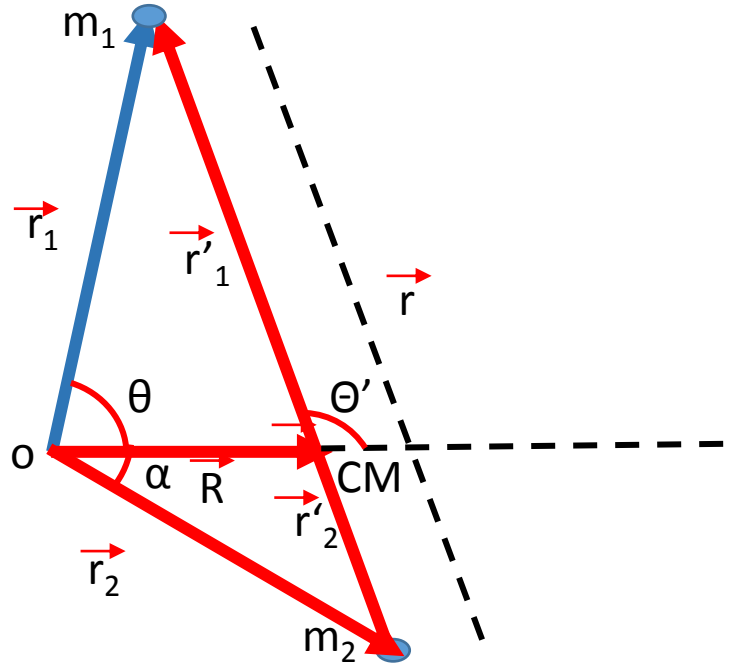
let...$\vec{r}_1'$ and..$\vec{r}_2'$...be..the..postion..vector..1..and..2..particle..their..C.M..in.frame

$\vec{r}_1 - \vec{r}_2 = \vec{r}$ Or..$\vec{r}_1' - \vec{r}_2' = \vec{r}$ ----()..relative..postion...of..vectors.1and..2..

$m_1\vec{r}_1' + m_2\vec{r}_2' = 0...(4)$

$\therefore \vec{r}_1' = -\frac{m_2}{m_1}\vec{r}_2'$ Or..$\vec{r}_2' = -\frac{m_1}{m_2}\vec{r}_1'$....(5)

$\vec{r}_1' - \vec{r}_2' = \vec{r} \therefore \vec{r}_2' = \vec{r}_1' - \vec{r} - - - (6)$



we..know..$m_1\vec{r}_1' + m_2\mathbf{r_2}' = 0$

$m_1\vec{r}_1' + m_2(\vec{r}_1' - \vec{r}) = 0..\therefore m_1\vec{r}_1' + m_2\vec{r}_1' - m_2\vec{r} = 0$

$\vec{r}_1'(m_1 + m_2) - m_2\vec{r} = 0..\vec{r}_1'(m_1 + m_2) = m_2\vec{r}$

$\vec{r}_1' = \left(\frac{m_2}{m_1 + m_2}\right)\vec{r}$..and..$\vec{r}_2' = -\left(\frac{m_1}{m_1 + m_2}\right)\vec{r}$.....(7)

$\vec{r}_1' = \frac{\mu}{m_1}\vec{r}$..and..$\vec{r}_2' = -\frac{\mu}{m_2}\vec{r}$, where$\mu = \left(\frac{m_1 m_2}{m_1 + m_2}\right.$

Re lative..velo..between..2..particles.$v = \left(\frac{dr}{dt}\right)$

velo...of..centra..of..mass.$V = \left(\frac{dR}{dt}\right.$

$\vec{u}_1$..*velo*...*of* ..1..*particle*...**Before...collistion**.*lab*.. *fream*

Before.Collistion.2..*particle*..*is*..*at*..*rest*...*relative*...*velo*..*v*..*is*.*equal*..*to*..$\vec{u}_1$

law..*of* ..*consevation*.*momentum*..

$(m_1 + m_2)\vec{V} = m_1 u_1$

$(m_1 + m_2)\vec{V} = m_1 v_1 (u_1 = v)$...(1)..*diff* ..*below*..*eq*..*w*.*r*.*to*..*time*

$\vec{r'_1} = \frac{\mu}{m_1}\vec{r}$..*and*..$\vec{r'_2} = -\frac{\mu}{m_2}\vec{r}$,..**where**$\boldsymbol{\mu} = \left(\frac{\mathbf{m_1 m_2}}{\mathbf{m_1 + m_2}}\right)$

Elastic Scattering Angles in Lab and central of mass system Systems

- The **Scattering angle** is the angle which deflected the object by particle when it come in contact with it.
- **we obtained the relation between velocities momentum and scattering angles in lab and C.M system basically**

$form..last..slid.fig....\vec{r}_1 = R + r\vec{}_1.....(1)$

$.....\vec{r}_2 = R + r_2\vec{}.....(2)$

$corresponding..final..velocities..$

$$\frac{d\vec{r}_1}{dt} = \frac{dR}{dt} + \frac{d\vec{r}'_1}{dt}\ Or..v_1 = V + v'_1.....(3)$$

$$\frac{d\vec{r}_2}{dt} = \frac{dR}{dt} + \frac{d\vec{r}'_2}{dt}\ Or..v_2 = V + v'_2....(4)$$

$The..final..momenta...of.the..particle..are..$

$$P_1 = m_1 v_1 = m_1 V + m_1 v'_2 = m_1 V + \mu v.....(5)$$

$$P_2 = m_2 v_2 = m_2 V + m_2 v'_2 = m_2 V - \mu v....(6)$$

$$r\vec{}_1 (m_1 + m_2) - m_2 \vec{r} = 0..r\vec{}_1 (m_1 + m_2) = m_2 \vec{r}$$

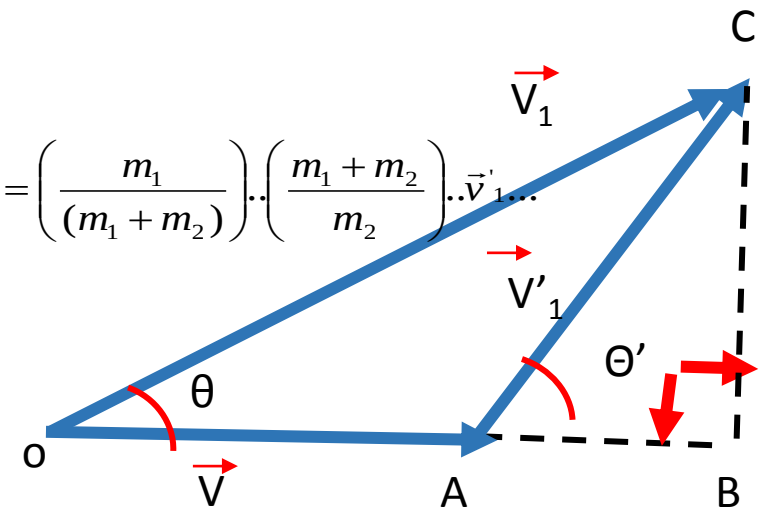
$from..fig..we..have..OB = v_1 \cos\theta..and..BC = v_1 \sin\theta = v'_1 \sin\theta',,and..AB = v'_1 \cos\theta'$

$$\tan\theta = \frac{BC}{OB} = \frac{BC}{OA + AB} = \frac{v'_1 \sin\theta'}{V + v'_1 \cos\theta'} = \frac{\sin\theta'}{\frac{V}{v'_1} + \cos\theta'}....(7)$$

$$(m_1 + m_2)\vec{V} = m_1 v_1 (u_1 = v)..we..know..V = \left(\frac{m_1}{(m_1 + m_2)}\right).v....(8)..\vec{v} = \frac{m_1}{\mu}\vec{v}'_1....\vec{v} = \left(\frac{m_1 + m_2}{m_2}\right)..\vec{v}'_1...V = \left(\frac{m_1}{(m_1 + m_2)}\right)..\left(\frac{m_1 + m_2}{m_2}\right)..\vec{v}'_1...$$

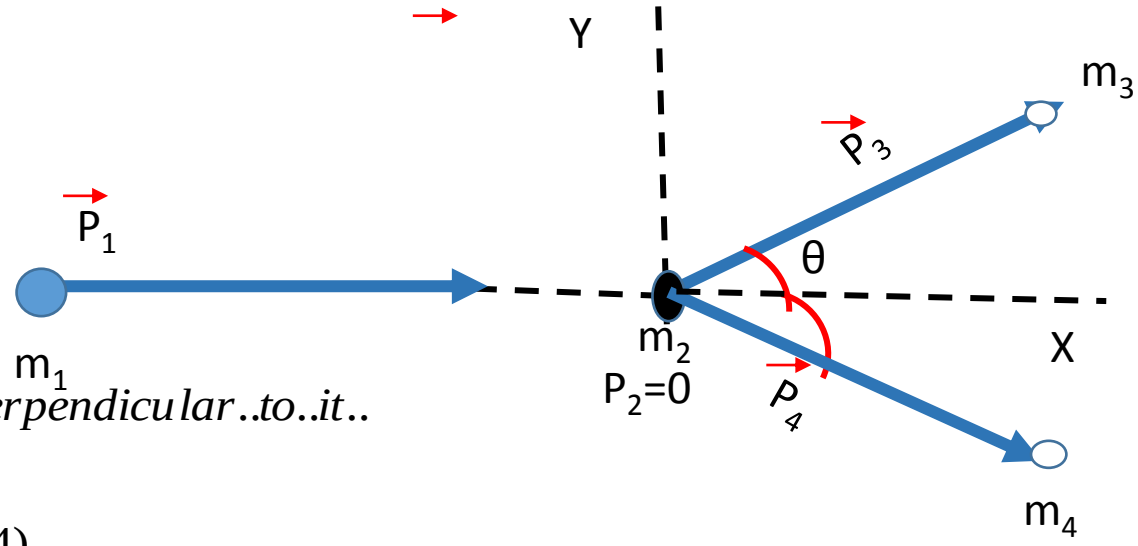
$$or..V = \left(\frac{m_1}{(m_2)}\right)..\vec{v}'_1...(9) sub..value..of..V..in..eq(7)..\tan\theta = \frac{\sin\theta'}{\frac{m_1}{m_2} + \cos\theta'}....(10)$$

$relation..between....\theta..and..\theta'...depends..on..ratio..\frac{m_1}{m_2}$



Inelastic Scattering

- The **Scattering angle** is the angle which deflected the object by particle when it come in contact with it.
- **we obtained the relation between velocities momentum and scattering angles in lab and C.M system basically**



$law..of\ ..consevation...linear\ ..momentum..and..energy,,$

$\vec{P}_1 = \vec{P}_3 + \vec{P}_4 ...(1)$

$\frac{1}{2}m_1\vec{v}_1 = \frac{1}{2}m_3\vec{v}_3 + \frac{1}{2}m_4\vec{v}_4 + Q..Or..\frac{\vec{P}_1^{\,2}}{2m_1} = \frac{\vec{P}_3^{\,2}}{2m_3} + \frac{\vec{P}_4^{\,2}}{2m_4} + Q ...(2)$

$Q..is..\text{int}\ erms..of\ ..scattering\ ..angle..\theta ...\therefore ..$

$we..have..to.use..consevation...momentum.along, incident\ ..and..perpendicu\,lar..to..it..$

$incident\ ..direction. \rightarrow P_1 = P_3\cos\theta + P_4\cos\phi .....(3)$

$perpendicu\,lar..to..incident\ ..direction.. \rightarrow 0 = P_3\sin\theta - P_4\sin\phi ...(4)$

$\therefore P_3\sin\theta = P_4\sin\phi ...(5)\ form..eq(3)...$

$P_1 - P_3\cos\theta = P_4\cos\phi ....(6) squaring..and..adding..(5)..and..(6)$

$P_1^{\,2} + P_3^{\,2} - 2P_1P_3\cos\theta = P_4^{\,2} ....(7) sub..value..of\ ..P_4..in..eq..2$

$$\frac{\vec{P}_1^{\,2}}{2m_1} = \frac{\vec{P}_3^{\,2}}{2m_3} + \frac{(P_1^{\,2} + P_3^{\,2} - 2P_1P_3\cos\theta)}{2m_4} + Q$$

$$Q = \frac{\vec{P}_1^{\,2}}{2m_1} - \frac{\vec{P}_3^{\,2}}{2m_3} - \frac{\vec{P}_1^{\,2}}{2m_4} - \frac{\vec{P}_3^{\,2}}{2m_4} + \frac{2P_1P_3\cos\theta}{2m_4} ..Or.. \frac{\vec{P}_1^{\,2}}{2m_1} - \frac{\vec{P}_3^{\,2}}{2m_3} - \frac{\vec{P}_1^{\,2}\times m_1}{m_1\times 2m_4} - \frac{\vec{P}_3^{\,2}\times m_3}{m_3\times 2m_4} + \frac{2P_1P_3\cos\theta}{2m_4}$$

$T_1 = \frac{\vec{P}_1^{\,2}}{2m_1} T_2 = \frac{\vec{P}_3^{\,2}}{2m_3} Or..P_1^{\,2} = 2m_1T_1 \therefore P_1 = \sqrt{2m_1T_1}$

$$\frac{\vec{P}_1^{\,2}}{2m_1}(1-\frac{m_1}{m_4}) - \frac{\vec{P}_3^{\,2}}{2m_3}(1-\frac{m_3}{m_4}) + 2\sqrt{\frac{T_1T_3m_1m_3}{m^2{}_4}}\cos\theta ..Or..Q = T_1(1-\frac{m_1}{m_4}) - T_2(1-\frac{m_3}{m_4}) + 2\sqrt{\frac{T_1T_3m_1m_3}{m^2{}_4}}\cos\theta .$$

$case(i) = if\ ..m_2 . \geq\geq .m_1 (\ proton - electron..scattering\ )$

$\frac{m_2}{m_1} \to .0.eq..(10)...becomes..\therefore \tan\theta = \frac{\sin\theta^{'}}{\cos\theta^{'}}..or.\tan\theta = \tan\theta^{'}$

$\theta = \theta^{'}..if\ ..mass.of\ .projecile..is..very..small.as.comp..to..mass.of\ .t\arg et$

$the..scattering\ ..angle..in..lab.\ frame...and..CM..\ ferme..are..same..$

$case(ii) = if\ ..m_1 . = .m_2 (\ proton - proton..scattering\ )$

$\frac{m_1}{m_2} = .1\tan\theta = \frac{\sin\theta^{'}}{1+\cos\theta^{'}} = \frac{2\sin(\theta^{'}/2)..\cos(\theta^{'}/2)}{2\cos^2\theta^{'}/2}$

$\tan\theta = \tan(\theta^{'}/2)..Or..\theta = (\theta^{'}/2)....if\ ..mass.of\ .projecile..is.equal..to..mass.of\ .t\arg et$

$the..scattering\ ..angle..in..lab.\ frame...is..equal..to..half\ ..of\ ..scattering\ ..angle...CM..\ ferme..$

$case(iii) = if\ ..m_1 . \geq\geq .m_2$

$\frac{m_1}{m_2} \to \infty.\tan\theta = 0..\theta = 0$

$Thedre..is..mostly...a..scattering\ ...in..the..\ forward..direction..$