Total No. of Questions : 4]

SEAT No. : ____

PA-2112

[Total No. of Pages : 3

[5901]-101

S.Y.B.Sc. (Regular)

MATHEMATICS

MT-231 : Calculus of Several Variables

(2019 Pattern) (Semester - III) (Credit System) (Paper-I) (23111)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following. **[5×1=5]**

a) Find the domain and range of $g(x,y)=\sqrt{g-x^2-y^2}$.

b) Where is the function

$$f(x,y)=\frac{x^2-y^2}{x^2+y^2}$$

continuous?

c) Find $\frac{\partial z}{\partial x}$ if $x^3+y^3+z^3+6xyz=1$.

d) Find the critical point of $f(x,y)=x^2+y^2-2x-6y+14$.

e) Write the condition for a critical point (a, b) of a function $f(x, y)$ to be a saddle point.

f) Evaluate

$$\int_0^1\int_0^3 e^{x+3y}\,dx\,dy$$

g) Find the Jacobian of the transformation $x = 5u–v$, $y = u + 3v$.

P.T.O.

Q2) a) Attempt any one of the following. **[5]**

i) If $f(x, y)$ is a function of two variables, write the formulas for $f_x(x,y), f_y(x,y), f_{xx}(x,y), f_{xy}(x,y)$ and $f_{yy}(x,y)$.

ii) State clairaut's theorem. Write laplace's equation in two dimension. Give an example of a function of two variables that satisfies laplace's equation.

b) Attempt any one of the following. **[5]**

i) Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

ii) Verify that the function $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of the three dimensional Laplace equation.

Q3) a) Attempt any one of the following. **[5]**

i) If $f(x, y)$ is a homogeneous function of degree n that has continuous second order partial derivatives, then show that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y).$$

ii) Explain the second derivative test to classify the critical points of a function of two variables into extreme points and saddle point.

b) Attempt any one of the following. **[5]**

i) If $u = x^4y + y^2z^3$, where $x = rse^t$, $y = rs^2e^{-t}$, and $z = r^2s \sin t$, find the value of $\frac{\partial u}{\partial s}$ when $r = 2$, $s = 1$, $t = 0$.

ii) A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box.

Q4) a) Attempt any one of the following. **[5]**

i) State Fubini's theorem. Write the formula for change of cartesian coordinates to polar coordinates in a double integral.

ii) Write the equations of relationship between rectangular coordinates (X, Y, Z) and the spherical coordinates (ρ, θ, ϕ). Hence find the rectangular coordinates of a point $\left(2, \frac{\pi}{2}, \frac{\pi}{2}\right)$ in spherical coordinates.

b) Attempt any one of the following. **[5]**

i) Evaluate

$$\int_1^2 \int_0^{2z} \int_0^{\ln x} xe^{-y}\, dy\, dx\, dz$$

ii) Evaluate

$$\int_0^1 \int_{3y}^{3} e^{x^2}\, dx\, dy$$

by reversing the order of integration.

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Total No. of Questions : 4]

SEAT No. : ____

PA-2113

[Total No. of Pages : 2

[5901]-102
S.Y. B.Sc.
MATHEMATICS (Paper - II)
MT-232 (A) : Numerical Methods and its Applications
(2019 Pattern) (CBCS) (Semester - III) (23112A)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Use of calculator is allowed.***

Q1) **Attempt any Five of the following :** **[5 × 1 = 5]**

a) Define absolute error.

b) Write the formula to find first approximation x_1 in Regula-Falsi method with initial approximations a and b.

c) Simplify E^3x^2 take $h = 1$.

d) Prove that $\nabla(k) = 0$ where k is constant.

e) Write the formula for $\frac{dy}{dx}$ at $x = x_n$ in terms of ∇.

f) Write Runge-Kutta second order formulae to solve $\frac{dy}{dx} = f(x, y)$ with initial condition $y(x_0) = y_0$.

g) Write the formula for $y_1^{(n)}$ in modified Euler's method.

Q2) a) **Attempt any ONE of the following :** **[5]**

i) Explain Bisection method to find root of $f(x) = 0$.

ii) Derive Lagrange's interpolation formula.

P.T.O.

b) **Attempt any ONE of the following :** **[5]**

i) Evaluate $\int_0^6 e^x dx$ by Simpson's $\frac{3}{8}$ rule using the data e = 2.71, e^2 = 7.38, e^3 = 20.08, e^4 = 54.59, e^5 = 148.41, e^6 = 403.42.

ii) Solve $\frac{dy}{dx} = -y$ with $y(0) = 1$ by Euler's method. Take $h = 0.01$ and obtain $y(0.01)$, $y(0.02)$, $y(0.03)$

Q3) a) **Attempt any ONE of the following :** **[5]**

i) Explain Taylor's series method to solve initial value problem.

ii) Explain modified Euler's method to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

b) **Attempt any ONE of the following :** **[5]**

i) Find the real root of $x^2 - 2x - 1 = 0$ between 1 and 3 by Regula-Falsi method (perform two iterations)

ii) Prove that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$.

Q4) a) **Attempt any ONE of the following :** **[5]**

i) Write the rules for round-off the number to significant figure.

ii) Derive the formula for $\frac{dy}{dx}$ at $x = x_0$ in terms of Δ.

b) **Attempt any ONE of the following :** **[5]**

i) Find $\sqrt[4]{74}$ by Newton-Raphson method. Perform two iterations.

ii) Find y when $x = 0.1$ by Runge-Kutta fourth order formula given $\frac{dy}{dx} = x + y$ with $y(0) = 1$.

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Total No. of Questions : 4]

SEAT No. :

PA-2114

[Total No. of Pages : 3

[5901]-103

S.Y. B.Sc.

MATHEMATICS (Paper - II(B))

MT-232(B) : Graph Theory

(2019 Pattern) (Credit System) (Semester - III) (23112B)

Time : 2 Hours] *[Max. Marks : 35*

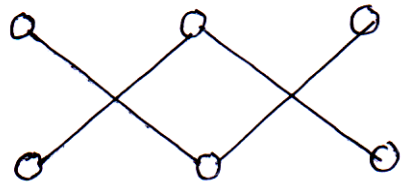
Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

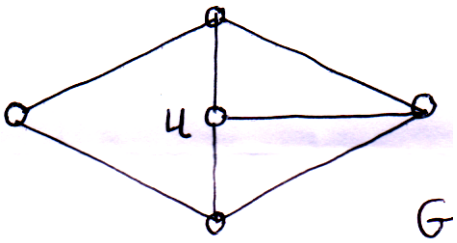
Q1) Attempt any FIVE of the following **[5]**

a) Define : Finite graph.

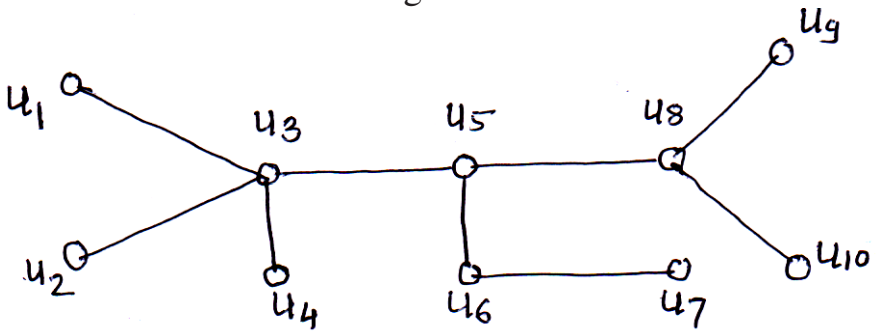
b) Determine the following graph is connected or not.



c) For the following graph G, Draw a graph G-u.



d) Find centre of the Following tree.

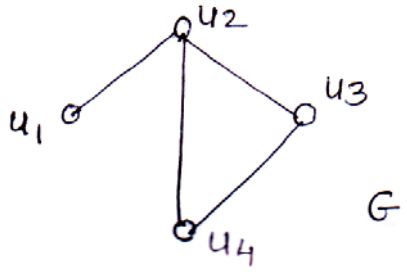


e) What is the value of prefix expression / 6 – 42?

P.T.O.

f) Find the edge connectivity of complete graph with n vertices.

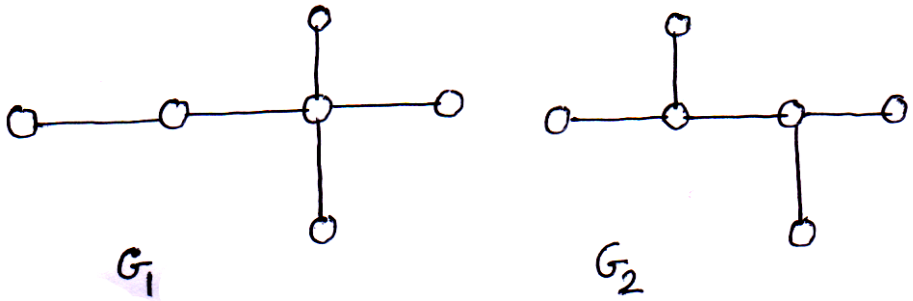
g) Find cut vertex in the following graph G.



Q2) a) Attempt any One of the following : **[5]**

i) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

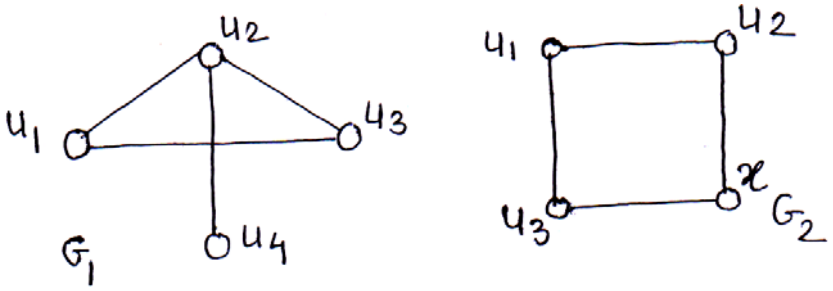
ii) Show that the following graphs are not isomorphic.



b) Attempt any One of the following : **[5]**

i) How many edges are there in a graph with 5 vertices with each of degree 8?

ii) Find $G_1 \cup G_2$ and $G_1 \cap G_2$ for the following graphs.



Q3) a) Attempt any ONE of the following : **[5]**

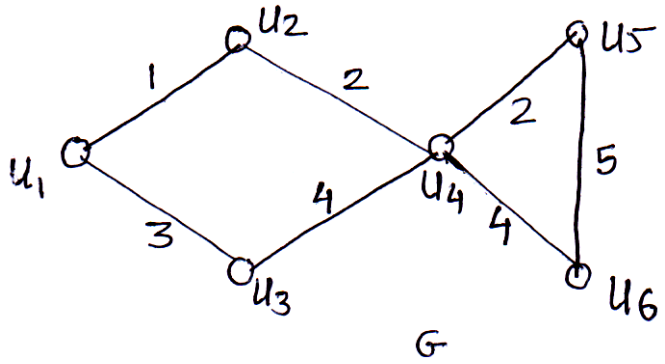
i) If G is a simple graph with n vertices and k components then prove that G can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

ii) In a tree T, prove that there is one and only one path between any two vertices.

b) Attempt any ONE of the following : **[5]**

i) Draw all simple non isomorphic graphs on 3 vertices.

ii) Using Krushkal's algorithm, obtain the shortest spanning tree in the graph G.



G

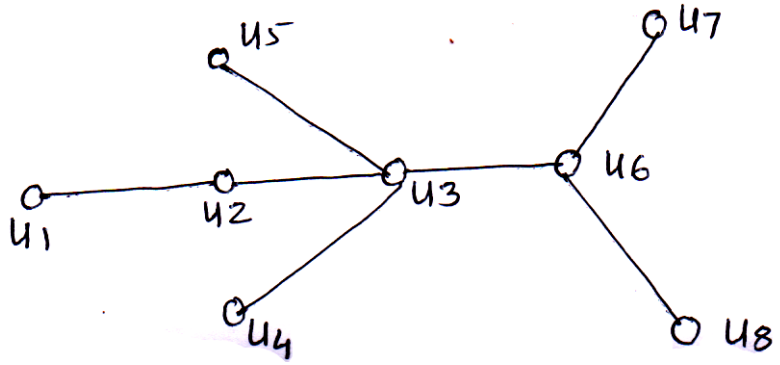
Q4) a) Attempt any ONE of the following : **[5]**

i) If T is a binary tree with n vertices then prove that number of pendant vertices in it is $\frac{n+1}{2}$.

ii) Prove that the edge connectivity of a connected graph G can not exceed the smallest degree of G i.e. $\lambda(G) \leq \delta(G)$, where $\delta(G)$ denotes the smallest degree in a graph G.

b) Attempt any One of the following : **[5]**

i) Find eccentricity of each vertex in the following tree. Also find centre, radius of tree.



ii) Is it possible to construct a graph on 11 vertices and 21 edges with edge connectivity 4? Justify.