Total No. of Questions : 4]

SEAT No. :

PA-2072

[Total No. of Pages : 2

[5901]-51
F.Y. B.Sc. (Mathematics)
MT-121 ANALYTICAL GEOMETRY
(2019 Pattern) (Semester - II) (Paper - I)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates :

1) ***All questions are compulsory.***
2) ***Figures to the right indicates full marks.***

Q1) **Attempt any Five of the following** : **[5 × 1 = 5]**

a) Show that the equation $4x^2 - 4xy + y^2 - 8x - 6y + 5 = o$ represents a parabola.

b) Find the direction cosines of line AB where A (2,3,-1) and B (0-1,2).

c) State the equation of plane in the intercept form.

d) Find the distance between parallel planes $x + 2y - 3z = 4$ and $2x + 4y - 6z = 11$.

e) Find the centre of the central conic $2x^2 - 2xy + 3y^2 + 6x - 4y - 1 = 0$

f) Find the equation of the sphere with center at (1,2,3) and radius 4.

g) State the direction cosines of X-axis.

Q2) **Attempt any One of the following** : **[5]**

a) i) Prove that the section of a sphere by the plane is a circle.

ii) Find the angle θ through which the axes are rotated so that the transformed form of the expression $ax^2 + 2hxy + by^2$ is free from the product term.

P.T.O.

b) **Attempt any One of the following** : **[5]**

i) What does the equation $3x^2 + 2xy + 3y^2 - 18x - 22y + 50 = 0$ become under rotation of the axes through 45º?

ii) Find an equation of the sphere on AB as a diameter where A (2,-3,1) and B (-1,-2,4).

Q3) **Attempt any One of the following** : **[5]**

a) i) Derive equation of the plane in normal form.

ii) Derive the relation between old and new co-ordinates when origin is shitted to the point (h,k).

b) **Attempt any One of the following** : **[5]**

i) Find the angle between two lines planes $2x - y + 2z + 1 = 0$ and $3x + 2y + 6z - 5 = 0$.

ii) Find the angle between two lines whose direction cosines are connected by the relations $\ell + 2m - 2n = 0, 3\ell m - mn = 0$

Q4) **Attempt any One of the following** : **[5]**

a) i) Derive equation of the plane passing through three points $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$

ii) Obtain the condition for given two lines. $\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and $\frac{x-\alpha'}{\ell'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$ tarbe coplanar.

b) **Attempt any One of the following** : **[5]**

i) Find the equation of tangent plane to the sphere $x^2 + y^2 + z^2 + 4x - 5y - 3z - 3 = 0$ at point (1 2,-1)

ii) Find the direction cosines of a line which is perpendicular to two lines having direction ratios 2,-1,2 and 3,6,2.

Total No. of Questions : 4]

SEAT No. :

PA-2073

[Total No. of Pages : 3

[5901]-52
F.Y. B.Sc.
MATHEMATICS
MT-122: Calculus - II
(2019 Pattern) (Semester - II) (Paper - II) (12112)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates :

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) **Attempt any Five of the following** : **[5 × 1 = 5]**

a) Does the absolute value function $f(x)=|x|, x\in\mathbb{R}$ differentiable at $x=0$? Justify.

b) State Rolle's Theorem.

c) Evaluate $\lim_{x\to 0}\frac{\tan tx}{x}$.

d) Solve $\frac{dy}{dx}=x(1+y^2)$.

e) State Taylor's theorem with Lagrange's form of remainder.

f) Check whether the following differential equation is exact or not.

$$6x^2y^2dx+4x^3ydy=0$$

g) Find an integrating factor for the differential equation,

$(x^2+y^2+2x)dx+2ydy=0$

P.T.O.

Q2) a) **Attempt any One of the following** : **[5]**

i) State and prove Lagrange's Mean Value Theorem.

ii) If $y = e^{ax}\sin(bx+c)$ then show that n[th] derivative of y is,

$y_n = r^n e^{ax}\sin(bx+c+n\theta)$, where $r = \sqrt{a^2+b^2}$ and $\theta \tan^{-1}\left(\frac{b}{a}\right)$

b) **Attempt any One of the following** : **[5]**

i) Find Maclaurin's series of $\sin x$ and hence show that

$x\sin x = x^2 - \frac{x^4}{3!} + \frac{x^6}{5!}$ --

ii) For the function $f:\mathbb{R}\to\mathbb{R}, f(x) = x^2 - 3x + 5,$ Find points of relative extrema and determine the intervals on which the function is increasing and decreasing.

Q3) a) **Attempt any One of the following** : **[5]**

i) State and prove the Leibnitz theorem for n[th] derivative of the product of two differentiable functions.

ii) If $p(x)$ is continuous on (a,b) then show that the general solution of the homogeneous equation, $\frac{dy}{dx} + P(x)y = 0$ on (a,b) is $y = ce^{-\phi(x)}$, where $\phi(x) = \int p(x)dx$.

b) **Attempt any One of the following** : **[5]**

i) Evaluate $\lim_{x\to\pi/2}(\sec x - \tan x)$

ii) Solve the following differential equation,
$(y\sin y)dx + x(\sin y - y\cos y)dy = 0$.

Q4) a) **Attempt any One of the following** : **[5]**

i) Explain the method of solving Bernoulli's equation, $\frac{dy}{dx} + p(x)y = f(x)y^n$ where n can be any real number other than 0 and 1.

ii) Let 'C' be an interior point of the interval I at which $f : I \to \mathbb{R}$ has a relative extremum. If the derivative of *f* at 'C' exists, then show that $f'(C) = 0$.

b) **Attempt any One of the following** : **[5]**

i) Find the general solution of the following equation by method of variation of parameters, $\frac{dy}{dx} + \left(\frac{1}{x} - 1\right)y = \frac{2}{x}$.

ii) Verify Cauchy's Mean Value Theorem for the functions $f(x) = x^3, g(x) = \tan^{-1}{}_x$ on [0,1], if so find the value of 'C' in the open interval (0,1).