Total No. of Questions : 4]

SEAT No. : ____

P4674

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[5822]-101

F.Y.B.Sc.

MATHEMATICS

MT-111 : Algebra

(2019 Pattern) (Semester - I) (11111)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following. **[5]**

a) Find g.c.d. of 35 and 49.

b) Define equivalence relation on z.

c) Let $f : R \to R$, $g : R \to R$ defined as $f(x) = x + 1$ and $g(x) = x^2$, find fog.

d) If w is a cube root of unity then find the value of $1 + w + w^2$.

e) Is $R_1 = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (2, 1), (2, 3)\}$ reflexive relation on the set $A = \{1, 2, 3, 4\}$? Justify?

f) Find the value of $\overline{100}$ in Z_3.

g) Find the modulus of $z = 1 + \sqrt{3}\hat{z}$.

Q2) A) Attempt any one of the following. **[5]**

i) State and prove De-Moivre's theorem for an integer indices.

ii) Prove that there are n distinct residue classes modulo n in integer.

P.T.O.

B) Attempt any one of the following. **[5]**

a) Find the g.c.d. 'd' of integers 357 and 2210 and express $d = 2210x + 357y$ for some $x, y \in z$.

b) Find the remainder of 7^{486} when divided by 13.

Q3) A) Attempt any one of the following. **[5]**

a) Prove that every partition of non empty set X defines an equivalence relation on X.

b) Prove that any two equivalence classes are either identical or disjoint.

B) Attempt any one of the following. **[5]**

a) If a, b, c are integers such that $a|bc$ and $(a, b) = 1$ then show that a|c.

b) Which elements of z_6 satisfies $x^2 = x$?

Q4) A) Attempt any one of the following. **[5]**

a) Let $z_1, z_2 \in c$ then prove that $|z_1 + z_2| \leq |z_1| + |z_2|$

b) State and prove Euclid's lemma.

B) Attempt any one of the following. **[5]**

a) Find the expression for $\cos^5\theta$ in terms of cosine of multiple of θ.

b) Express $z = 1 + i$ in polar form.



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SEAT No. :

P4675

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[5822]-102
F.Y. B.Sc.
MATHEMATICS
MT - 112 : Calculus - I
(CBCS) (2019 Pattern) (Semester - I) (Paper - II) (11112)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following: **[5]**

a) Determine the set $A = \left\{ x \in \mathbb{R} \,/\, x < \frac{1}{x}, x > 0 \right\}$.

b) Find first four terms of the sequence (x_n) where $x_n = \frac{1}{n(n+1)}$.

c) Define monotone sequence of real numbers.

d) Find domain of the function $f(x) = \frac{1}{x-3}$.

e) State boundedness theorem for continuous function on an interval.

f) At what points the function $f(x) = \frac{x}{(x-2)(x-4)}$ is discontinuous?

Q2) a) Attempt any one of the following: **[6]**

i) State and prove triangle inequality for real numbers. Hence, prove that $|a-b| \geq ||a|-|b||$, for all $a,b \in \mathbb{R}$.

ii) Define limit of sequence of real numbers and prove that if limit of sequence exists then it is unique.

P.T.O.

b) Attempt any <u>one</u> of the following: **[4]**

i) Evaluate : $\lim_{n \to \infty} \frac{2n^2+3}{5n^2+1}$.

ii) Discuss the continuity of the function $f(x)$ at origin,

where $f(x) = \frac{e^{1/x}}{1+e^{1/x}}$; for $x \neq 0$

$= 0$; for $x = 0$

Q3) a) Attempt any <u>one</u> of the following: **[6]**

i) Show that the sequence (x_n) defined by $x_1=1$ and $x_{n+1} = \sqrt{2+x_n}, \forall n \in N$ is monotonic and bounded.

ii) Find the values of α and β, if the function $f(x)$ is continuous in (–3, 3) where, $f(x) = x + \alpha$; $-3 < x < 0$

$= 2x + 1$; $0 \leq x > 2$

$= \beta - x$; $2 \leq x < 3$

b) Attempt any <u>one</u> of the following: **[4]**

i) Find the infimum and supremum of the set $S = \left\{1 - \frac{(-1)^n}{n} / n \in N\right\}$.

ii) Let $f : A \to \mathbb{R}$ and c be the cluster point of A. If $\lim_{x \to c} f(x) = L$ and the sequence (x_n) converges to 'c' such that $x_n \neq c$, for all $n \in N$ then prove that $(f(x_n))$ converges to L.

Q4) a) Attempt any <u>one</u> of the following: **[6]**

i) State squeeze theorem and hence show that $\lim_{x \to 0} x \cdot \cos\left(\frac{1}{x}\right) = 0$.

ii) Let $f : I \to \mathbb{R}$ be continuous on I = [a, b]. If $K \in \mathbb{R}$ satisfies $f(a) < K < f(b)$ then prove that there exist a point $c \in I$ between a and b such that $f(c)$=K.

b) Attempt any <u>one</u> of the following: **[4]**

i) If $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ has limit at $x = c \in \mathbb{R}$, then prove that f is bounded on some neighbourhood of 'c'.

ii) Let $A \subseteq \mathbb{R}$ and $f, g : A \to \mathbb{R}$. If f and g are continuous functions at $c \in A$ then prove that $f+g$ and $f \cdot g$ are also continuous functions at $x = c$.