# The Contribution of Srinivasa Ramanujan

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Srinivasa Ramanujan was one of India's greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series.

Ramanujan was born in his grandmother's house in Erode, a small village about 400 km southwest of Madras (now Chennai). When Ramanujan was a year old his mother took him to the town of Kumbakonam, about 160 km nearer Madras. His father worked in Kumbakonam as a clerk in a cloth merchant's shop. In December 1889 he contracted smallpox.

When he was nearly five years old, Ramanujan entered the primary school in Kumbakonam although he would attend several different primary schools before entering the Town High School in Kumbakonam in January 1898. At the Town High School, Ramanujan was to do well in all his school subjects and showed himself an able all round scholar. In 1900 he began to work on his own on mathematics, summing geometric and arithmetic series.

Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quartic. The following year, not knowing that the quintic could not be solved by radicals, he tried (and of course failed) to solve the quintic.

It was in the Town High School that Ramanujan came across a mathematics book by G S Carr called *Synopsis of elementary results in pure mathematics*. This book, with its very concise style, allowed Ramanujan to teach himself

mathematics, but the style of the book was to have a rather unfortunate effect on the way Ramanujan was later to write down mathematics since it provided the only model that he had of written mathematical arguments. The book contained theorems, formulae and short proofs. It also contained an index to papers on pure mathematics which had been published in the European Journals of Learned Societies during the first half of the 19th century. The book, published in 1886, was of course well out of date by the time Ramanujan used it.

By 1904 Ramanujan had begun to undertake deep research. He investigated the series and calculated Euler's constant to 15 decimal places. He began to study the Bernoulli numbers, although this was entirely his own independent discovery.

$$\sum\left(\frac{1}{n}\right)$$

Ramanujan, on the strength of his good school work, was given a scholarship to the Government College in Kumbakonam which he entered in 1904. However the following year his scholarship was not renewed because Ramanujan devoted more and more of his time to mathematics and neglected his other subjects. Without money he was soon in difficulties and, without telling his parents, he ran away to the town of Visakhapatnam about 650 km north of Madras. He continued his mathematical work, however, and at this time he worked on hypergeometric series and investigated relations between integrals and series. He was to discover later that he had been studying elliptic functions.

In 1906 Ramanujan went to Madras where he entered Pachaiyappa's College. His aim was to pass the First Arts examination which would allow him to be admitted to the University of Madras. He attended lectures at Pachaiyappa's College but became ill after three months of study. He took the First Arts examination after having left the course. He passed in mathematics but failed all his other subjects and therefore failed the examination. This meant that he could not enter the University of Madras. In the following years

he worked on mathematics developing his own ideas without any help and without any real idea of the then current research topics other than that provided by Carr's book.

Continuing his mathematical work Ramanujan studied continued fractions and divergent series in 1908. At this stage he became seriously ill again and underwent an operation in April 1909 after which it took him some considerable time to recover. He married on 14 July 1909 when his mother arranged for him to marry a ten year old girl S Janaki Ammal. Ramanujan did not live with his wife, however, until she was twelve years old.

Ramanujan continued to develop his mathematical ideas and began to pose problems and solve problems in the *Journal of the Indian Mathematical Society*. He developed relations between elliptic modular equations in 1910. After publication of a brilliant research paper on Bernoulli numbers in 1911 in the *Journal of the Indian Mathematical Society* he gained recognition for his work. Despite his lack of a university education, he was becoming well known in the Madras area as a mathematical genius.

In 1911 Ramanujan approached the founder of the Indian Mathematical Society for advice on a job. After this he was appointed to his first job, a temporary post in the Accountant General's Office in Madras. It was then suggested that he approach Ramachandra Rao who was a Collector at Nellore. Ramachandra Rao was a founder member of the Indian Mathematical Society who had helped start the mathematics library.

Ramanujan sailed to India on 27 February 1919 arriving on 13 March. However his health was very poor and, despite medical treatment, he died there the following year.

The letters Ramanujan wrote to Hardy in 1913 had contained many fascinating results. Ramanujan worked out the Riemann series, the elliptic integrals, hypergeometric series and functional equations of the zeta function. On the other hand he had only a vague idea of what constitutes a mathematical proof. Despite many brilliant results, some of his theorems on prime numbers were completely wrong.

Ramanujan independently discovered results of Gauss,

Kummer and others on hypergeometric series. Ramanujan's own work on partial sums and products of hypergeometric series have led to major development in the topic. Perhaps his most famous work was on the number p(n) of partitions of an integer n into summands. MacMahon had produced tables of the value of p(n) for small numbers n, and Ramanujan used this numerical data to conjecture some remarkable properties some of which he proved using elliptic functions. Others were only proved after Ramanujan's death. In a joint paper with Hardy, Ramanujan gave an asymptotic formula for $p(n)$. It had the remarkable property that it appeared to give the correct value of $p(n)$, and this was later proved by Rademacher.

Ramanujan left a number of unpublished notebooks filled with theorems that mathematicians have continued to study. G N Watson, Mason Professor of Pure Mathematics at Birmingham from 1918 to 1951 published 14 papers under the general title *Theorems stated by Ramanujan* and in all he published nearly 30 papers which were inspired by Ramanujan's work. Hardy passed on to Watson the large number of manuscripts of Ramanujan that he had, both written before 1914 and some written in Ramanujan's last year in India before his death.

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